Numerical Analysis of Darcy-Forchheimer Flow and Heat Transfer over a Stretching Sheet with Uniform Heat Source

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Abstract

The analysis is made to explore the Darcy-Forchheimer flow and heat transfer of Maxwell fluid over a vertical stretching sheet uniform heat source/sink. Using similar transformations, the governing partial differential equations (PDEs) are converted into non-linear ordinary differential equations (ODEs). The resulting ODEs are solved numerically by Runge-Kutta fourth order method along with shooting technique. The outcomes of relevant parameters on velocity, temperature as well as skin friction coefficient and local Nusselt number are illustrated through graphs and tables. It is found that local inertia parameter, which is responsible for inertia drag, reduces the fluid velocity but adverse effect is observed on temperature field.

Keywords: *Darcy-Forchheimer model, heat transfer, vertical stretching sheet, uniform heat source/sink.*

1. Introduction

The flow problem in the boundary layer induced by a continuously moving or stretching surface is important in many manufacturing processes. In industry, polymer sheets and filament are manufactured by continuous extrusion of the polymer from a die to a windup roller, which is located at a finite distance away from a die. The thin polymer sheet constitutes a continuously moving surface with a non-uniform velocity through an ambient fluid or fluid with some prescribed velocity. Crane [1] examined the boundary layer flow caused by stretching sheet.

The flow and heat transfer through a porous media have received much attention in past few years due to its various practical applications in geothermal and oil reservoir engineering as well as other geophysical and astrophysical studies. Nayak et al. [3] have studied viscoelastic fluid over a stretching sheet. Dessie and Kishan [4] have incorporated viscous dissipation and heat source/sink on heat transfer flow of a fluid over stretching sheet. Swain et al. [5] have studied the effect of variable viscosity and thermal conductivity on MHD heat and mass transfer over a stretching sheet. Mahdy and Chamkha [6] have considered viscous dissipation and chemical reaction on mixed convective Darcy-Forchheimer fluid flow. All these studies have been confined in a porous medium.

A new dimension is added to the study of flow and heat transfer in a viscous fluid over a stretching surface by considering the effect of thermal radiation. Thermal radiation effect might play a significant role in controlling heat transfer process in polymer processing industries. The quality of the final product depends to a great extent on the heat controlling factors. The knowledge of radiative heat transfer in the system can perhaps lead to a desired product with a sought characteristic. Many processes in engineering area occur at high temperature and knowledge of radiation heat transfer becomes very important for the design of the pertinent equipment. Nuclear power plants, gas turbines and various propulsion devices for aircraft, missiles, satellites and space vehicles are the examples of such engineering areas. Makinde [7] have examined the effect of thermal radiation past a moving vertical porous plate. Golafshan and Rahimi [8] have studied the effect of radiation on mixed convection stagnation point flow of nanofluid over a vertical stretching sheet. Slip flow analysis on MHD nanofluid over a vertical stretching sheet with higher order chemical reaction is carried out by Swain et al. [9]. Hayat et al. [10] analyzed the heat and mass transfer of Darcy-Forchheimer flow with chemical reaction. Bakar et al. [11] studied the stagnation point flow in Darcy-Forchheimer porous medium over a shrinking sheet with slip boundary conditions. Uddin et al. [12] analyzed numerically the Darcy-Forchheimer flow of Sisko nanomaterial with nonlinear thermal radiation. Rasool et al. [13] carried out MHD Darcy-Forchheimer nanofluid flow over nonlinear stretching sheet.

The objective of this study is to put up a mathematical model for Darcy-Forchheimer boundary layer flow and heat transfer past a vertical stretching sheet with non-uniform heat source/sink. The governing PDEs are transferred into non-linear ODEs by using similarity transformations and then solved by Runge-Kutta fourth order method along with shooting technique. The effects of different parameters are presented through graphs and illustrated in details. Further, the shearing stress and the rate of heat transfer at the plate have been computed via tables and discussed in details.

2. Mathematical Formulation

We consider a steady two-dimensional boundary layer flow and heat transfer over a vertical stretching sheet in the presence of uniform heat source/sink. The flow is assumed to be in the *x*-direction, which is chosen along the sheet and the y-axis perpendicular to it. Let u and v are the tangential and normal velocities of the fluid respectively. The differential equations of fluid motion are based on Forchheimer which accounts for the drag exerted by the porous media, in the study of porous media flow analysis. The governing boundary layer equations for momentum and energy under Boussinesq's approximation are:

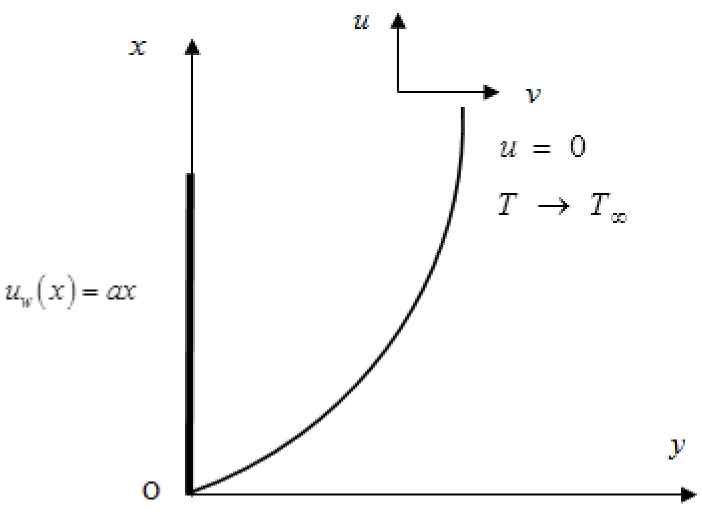

Fig. 1 Flow model and coordinate system

Continuity equation:

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0 \tag{1}$$

Momentum equation:

$$u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=\upsilon\frac{\partial^2 u}{\partial y^2}-\frac{\sigma B_0^2}{\rho}u-\frac{\upsilon}{Kp^*}u-\frac{c_b}{\sqrt{Kp^*}}u^2 \tag{2}$$

Equation of energy:

$$u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}=\frac{k}{\rho c_p}\frac{\partial^2 T}{\partial y^2}+\frac{Q^*}{\rho c_p}\left(T-T_\infty\right) \tag{3}$$

The corresponding conditions are

$$\begin{aligned}&\mathrm{y}=0: u=u_w\left(x\right)=ax, v=0, -k\frac{\partial T}{\partial y}=h\left(T_w-T_\infty\right)\\ &y\to\infty: u=0, T\to T_\infty\end{aligned} \tag{4}$$

where u,v are velocity components in x and y directions respectively, B_0 is the magnetic field strength, υ is the kinematic viscosity, σ is the electrical conductivity, ρ is the density, k is the thermal conductivity, T is the temperature, T_∞ is the ambient temperature of the fluid, c_p is the specific heat, c_b is the drag coefficient, Q^* is the heat source/sink coefficient, Kp^* is the permeability of the medium, $a(>0)$ is a constant, and h is the heat transfer coefficient .

By using the following similarity transformations and non-dimensional variables

$$\eta=\sqrt{\frac{a}{\upsilon}}y, u=axf'\left(\eta\right), v=-\sqrt{\upsilon a}\,f\left(\eta\right), \theta\left(\eta\right)=\frac{T-T_\infty}{T_w-T_\infty}$$

the equations (1) – (4) can be written as

$$f'''-\left(1+Fr\right)f'^2+ff''-\left(M+Kp\right)f'=0 \tag{5}$$

$$\frac{1}{\Pr}\theta''+f\theta'+Q\theta=0 \tag{6}$$

$$\left.\begin{array}{l}\eta = 0: f(0) = 0, f'(0) = 1, \theta'(0) = -Bi\left[1-\theta(0)\right] \\ \eta \to \infty: f'(\infty) \to 0, \theta(\infty) \to 0\end{array}\right\} \tag{7}$$

where $M = \frac{\sigma B_0^2}{a\rho}$ is the magnetic parameter, $Kp = \frac{aKp^*}{\upsilon}$ is the porosity parameter, $Fr = \frac{c_b}{x\sqrt{Kp^*}}$ is the local inertia parameter, $\Pr = \frac{\mu c_p}{k}$ is the Prandtl number, $Q = \frac{Q^*}{a\rho c_p}$ is the heat source/sink parameter, and $Bi = \frac{h}{k}\sqrt{\frac{\upsilon}{a}}$ is the Biot number.

The skin friction coefficient (C_f) and local Nusselt number (Nu_x) are given by $C_f = \frac{2\tau_w}{\rho u_w^2}$ and $Nu_x = \frac{xq_w}{k(T_w - T_\infty)}$ respectively.

Here, wall shear stress $\tau_w = \mu\left(\frac{\partial u}{\partial y}\right)_{y=0} \Rightarrow C_f\sqrt{\mathrm{Re}_x} = -f''(0)$ and wall heat flux $q_w = -k\left(\frac{\partial T}{\partial y}\right)_{y=0} \Rightarrow \frac{Nu_x}{\sqrt{\mathrm{Re}_x}} = -\theta'(0)$ where $\mathrm{Re}_x = \frac{ax^2}{\upsilon}$ is the local Reynolds number.

3. Results and Discussion

The coupled, non-linear PDEs (5) - (7) are solved by Runge-Kutta fourth order method with shooting technique using MATLAB software with step length $\Delta\eta = 0.01$ and the error tolerance 10^{-5}. The validation has been accomplished by comparing the present study with that of Khan and Pop [14] as particular case assigning $M = Fr = Kp = Bi = 0$ as shown in Table 1. During discussion, we have fixed the non-dimensional parameters as $M = 0.1, Kp = 0.5, Fr = 1, Q = Bi = 0.1$ and $\Pr = 2$ unless otherwise the values are mentioned.

Table 1 Comparison of the values of $-\theta'(0)$

	$-\theta'(0)$	
Pr	Khan and Pop [14]	Present study
0.07	0.0663	0.064295
0.2	0.1691	0.168294
0.7	0.4539	0.451590
2	0.9113	0.910680
7	1.8954	1.894921
20	3.3539	3.353507
70	6.4621	6.461822

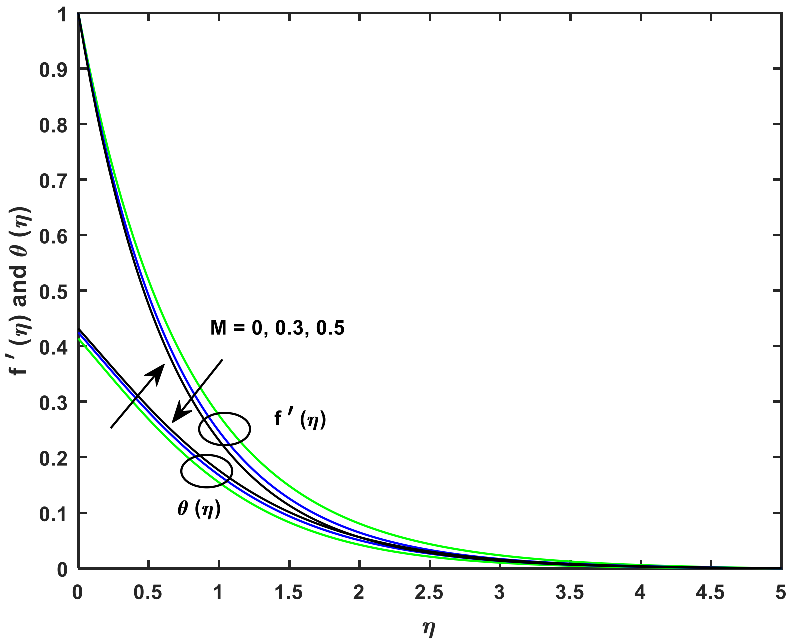

Fig. 2 Influence of M on velocity and temperature profiles

Fig. 2 shows the effect of magnetic parameter (M) on velocity $f'(\eta)$ and temperature $\theta(\eta)$ profiles. An increase in magnetic parameter causes to produce a Lorentz force which works in the opposite direction of the fluid flow. This force has tendency to slow down the motion of the fluid flow and consequently, produce more heat in the fluid. Therefore, an increase in M is to decrease the velocity and increase the temperature of the fluid in the flow domain.

Fig. 3 is depicted to study the influence of local inertia parameter (Fr) on velocity and temperature profiles in presence $(Kp = 0.5)$ and absence $(Kp = 0)$ of porous matrix. It is elucidated that on increasing in (Fr) the fluid velocity decreases. In case of porous spaces with bigger pore sizes, Forchheimer number accounts for the inertia effects due to porous medium and pressure drop disturbed by fluid-solid interaction which dominates the viscous interference. Thus, an increase in local inertia parameter causes a greater resistance to the flow; hence, the fluid velocity getting decreased whereas enhances the temperature of the fluid since more heat is generated due to porous medium.

Fig. 4 displayed to analyze the impact of heat source/sink parameter (Q) on temperature distribution. It is obvious from this figure that thickness of the thermal boundary layer increases on increasing heat source/sink parameter (Q).

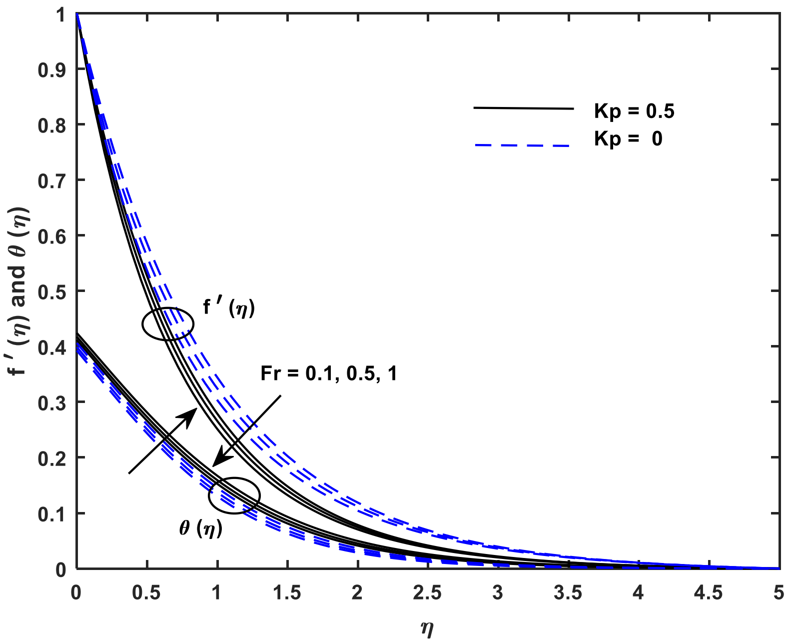

Fig. 3 Influences of Fr and Kp on velocity and temperature profiles

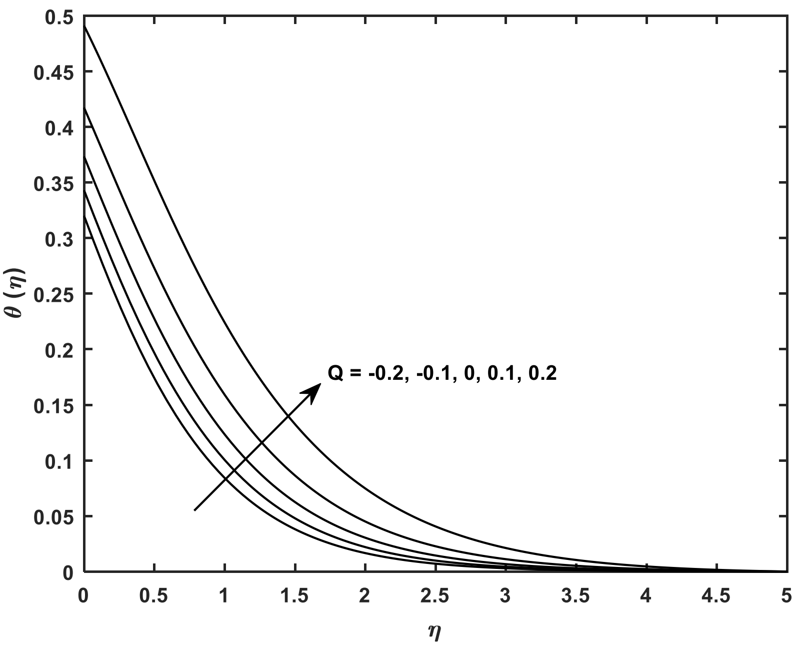

Fig. 4 Influence of Q on temperature profile

Table 2 Values of skin friction coefficient $-f''(0)$ and Nusselt number $-\theta'(0)$

when $Kp = 0.5$ and $\Pr = 2$.

M	Fr	Bi	Q	$-f''(0)$	$-\theta'(0)$
0.1	0.1	0.1	0.1	1.290355	0.087724
0.5				1.620293	0.095979
1.0				1.735964	0.096853
	0.5			1.780571	0.094248
	1.0			1.928377	0.093136
		0.3		1.928348	0.272901
		0.5		1.928347	0.367355
			0.3	1.919284	0.343609
			0.5	1.901410	0.308226

Table 2 is computed to observe the impacts of important physical parameters M, Fr, Bi and Q on skin friction coefficient and Nusselt number. The skin friction coefficient is getting enhanced on increasing values of M and Fr whereas slightly decreases with higher values of Q. On the other hand, local Nusselt number is getting enhanced on increasing values of M and Bi whereas decreases on increasing either of Fr and Q. These results are well supported by Seth et al. [15]

4. Concluding remarks

The present article focuses the effect of uniform heat source/sink on magnetohydrodynamic flow and heat transfer of an electrically conducting fluid over a vertical stretching sheet in a non-Darcy porous medium. The equations of the stated flow are solved numerically by effective shooting technique and graphs are plotted using obtained numerical values. We have compared the present study results with the existing results and they are in good agreement with the previous results. The following conclusions can be drawn from the present study:

- Increasing values of magnetic parameter declines the velocity of the fluid but enhances the thermal resistance that leads to boost the temperature profile.
- The local inertia parameter (Forchheimer parameter), which is responsible for inertia drag, reduces the fluid velocity but adverse effect is observed on temperature field.
- Biot number and Heat source/sink parameter have an increasing effect on temperature distribution.
- Magnetic parameter and local inertia parameter have same effects on skin friction coefficient but opposite effects on local Nusselt number.

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